Sparse-matrix algorithms for global eigenvalue problems

By P. Luchini, F. Giannetti, J. Pralits

DIMEC, Università di Salerno, Italy

luchini@unisa.it

The linear stability of two-dimensional and three-dimensional flows is, from an abstract mathematical viewpoint, an eigenvalue problem like any other. However, its numerical resolution by full-matrix QR algorithms like those found in LAPACK requires the handling of exceptionally large matrices; for this reason, researchers have generally turned their attention to sparse-matrix methods, the best known example of which is the implicitly restarted Arnoldi algorithm as implemented by Sorensen in the publicly available ARPACK library. These too have drawbacks, however, since for performance reasons a shift-and-invert preconditioner is practically always necessary which itself contains a costly matrix inversion. For this reason a third approach that is still popular is to euristically chase a critical Reynolds number by looking for neutral growth in a time-accurate numerical simulation; this is straightforward for anybody who already has a time-accurate code available, but wasteful of iterations because the time step is constrained by accuracy and not just by numerical stability; in addition it only produces one leading eigenvalue.

For the last three-four years, we in University of Salerno have been developing our own iterative algorithms, based on the requirement that the algorithm should not contain any matrix inversion, not even as a preconditioner, and should provide the leading few eigenvalues and eigenvectors of the stability problem with an easily parallelizable code structure that resembled as much as possible that of a time iteration. Some early results of this development were presented at the Fifth Conference on Bluff Body Wakes and Vortex-Induced Vibrations, Costa do Sauípe, Brazil, 12-15 Dec. 2007.

This presentation will start with a general overview of the classical eigenvalue algorithms, which underlie both the publicly available libraries and our own code, and then move on to the choices we made and the degree of success we have met using a simple subspace iteration. Subspace iteration is a more primitive algorithm then the implicitly restarted Arnoldi code (in fact, it can be considered its ancestor), but can more easily be combined with an imperfect matrix inversion, and is inherently more parallel. Already in 2007 we could combine subspace iteration with an immersed-boundary multigrid algorithm and presented some applications to the oscillations of a freely moving cylinder. Here we shall present an ampler range of applications, as well as discuss the still ongoing development of a possibly faster algorithm.

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